“Dynamic Response of Beam Under Moving Mass”

A THESIS SUBMITTED IN PARTIAL FULFILLMENT
OF THE REQUIREMENTS FOR THE DEGREE OF

Bachelor of Technology

In Mechanical Engineering

By

Mohan Charan Sethi
Roll No: 108ME078

Department of Mechanical Engineering

National Institute of Technology Rourkela

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Under the Guidance of

Dr. R. K. BEHERA

Department of Mechanical Engineering

National Institute of Technology Rourkela

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NATIONAL INSTITUTE OF TECHNOLOGY

ROURKELA

CERTIFICATE

This is to certify that the thesis entitled, "**DYNAMIC RESPONSE OF BEAM UNDER MOVING MASS**" by **Mr. Mohan Charan Sethi** in partial fulfillment of the requirements for the award of Bachelor of Technology Degree in Mechanical Engineering at the National Institute of Technology, Rourkela is an authentic work carried out by him under my supervision and guidance.

To the best of my knowledge the matter embodied in the thesis has not been submitted to any other University/ Institute for the award of any Degree or Diploma.

Date**:**

Dr.R.K.Behera

Dept. of Mechanical Engineering

National Institute of Technology

Rourkela-769008

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Mohan Charan Sethi

Roll No. 108ME078

Department of Mechanical Engineering

National Institute of Technology

Rourkela

ABSTRACT

This thesis is about the experimental study of linear dynamic response a structure (Euler- Bernoulli beam) under a moving mass.The moving mass is constant in magnitude and moves at constant velocity. The dynamic analysis of beam is done by taking two different cases. In the first case effects of inertia of the load were neglect and in second case effects of the beam inertia were also not considered. The study of the moving load is a great importance in the field of transportation. Examples of the structural elements which support the moving masses are bridges, guide ways, overhead cranes, cable ways, railway, roadways, runways, tunnels and pipelines etc.in the first phase, dynamic equation of motion with the moving load for simply supported beam is solved by analytical method and then experimental analysis is given. In the present work, effect of moving mass on both simply supported and cantilever beam for two different materials is investigated. The effect of variation of parameters on dynamic response of Euler- Bernoulli beam is studied. The beam is divided into 20 stations and the deflection of beam at mid-span is recorded while mass moves at constant velocity through different stations. The deflection at mid-span for different station is taken with the help of oscilloscope. The result obtained is plotted in the form of graphs for different velocities of mass.

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Nomenclature

E = Young's Modulus of the Respective Material

I = Second Moment of Area of the Beam Cross-Section

m = Mass per unit length

x = axial co-ordinate

t = time

W (x, t) or $u_z(x, t)$= the transverse deflection of the beam

F = applied force

u = position of the mass on the beam

δ (x-u) = Dirac Delta function

M = weight of the moving mass

g = gravitational acceleration

β = transverse displacement of the mass

G (x, u) = Dynamic green function

ω = circular frequency

β_n= frequency ratio

v = speed of the moving mass

L = length of the beam

CHAPTER 1

INTRODUCTION

1 INTRODUCTION

The moving load problem is a unavoidable difficulty in structural dynamics. A lot of hard work has been accounted during the last 100 years conducting with the dynamic response of railway bridges and later on highway bridges under the effect of moving loads. Beam type structures are widely used in many fields of engineering like civil, mechanical and aerospace.

Vehicle-bridge interaction is a vast area of interest in moving load problem. If the relative speeds involved are very low with respect to the critical speed then the problem can be framed as conventional and non-moving moving load problems. Treating them as a moving load problem involves more advanced mathematics and computation.

The dynamic effect of moving loads was not known until mid-19th century. When the Stephenson's bridge across river Dee Chester in England in 1947 collapsed, it motivates the engineers for research of moving load problem. In the past, the moving load problems are about railway bridges excited by travelling trains with moving structures travel in straight line. Taking an example of computer disc drive system where the magnetic reader or writer head maintain a moving load in circumferential direction and adopts circular path. The most general type of moving loads in structural studies is a constant or harmonic pure force.so it is said that a structure under moving pure load is equal to non-moving load vibration problem.so a moving load problem can be defined by using beam subjected to moving point wise mass which is also called moving mass problem.

The engineering structures in most case subjected to time and space varying loads. Because of increase in traffic intensity and speed, we need to study the structures in many aspects than earlier studies.in this paper the simple moving mass problem is represented with a beam over which concentrated load is moving and is theoretically represented in the form of a fourth order partial differential equation.it is solved analytically by separation of variable method and then experimentally analyzed the effect of different parameters.

Advance in transport technology and automobile engineering resulting high speed and heaviness of vehicles and other moving bodies' as a result corresponding structures have been subjected to vibration and dynamic stress much higher than ever before. This experimental work is concerned with the dynamic behavior of an Euler-Bernoulli beam subjected to a moving load.

This problem has significant attention in civil and mechanical engineering. The dynamic analysis of the vibrating beam is done by neglecting the disconnection of the moving mass from the beam during the motion and result is given by considering mass moving at constant speed and in one direction. Once the load departs from the beam, it begins to vibrate at in free vibration mode. Hence this process no longer comes within the scope of the experiment.

CHAPTER 2

LITRATURE REVIEW

2. LITRATURE REVIEW

The dynamic analysis of beam structure with beam structure with moving load is a fundamental problem in structural dynamics in the comparison to other dynamic load, the moving load very in position and that is why the moving load problem is a special topic in structural dynamics. Since 19^{th} century the moving load problem has become more dynamic in nature due to increased vehicle speed and structural flexibility. This trend also gives a significant value to structure-vehicle interaction phenomenon.

Many structures are designed to support moving masses such as bridges, guide ways, overhead cranes, cable ways, rails, roadways, tunnels, and pipeline etc. Also many members can be modeled as beams under moving loads in the design of machining processes. Vibration analysis of structure become a interesting area of research since 1897, when the Chester rail bridge collapsed in England [1].various kinds of moving load problem associated with structural dynamics have been presented in excellent monograph by Fryba [2].Load and an associated mass travelling over a massless beam is applicable to railway bridges and is solved by stokes [3].

Many analytical methods are given in past to solve simple moving load problem. For general analysis numerical method is used. Complex dimensional structures such as non-uniform beams subjected to a moving load was first treated by Kolousek [4, 5] who solved the most of the typical problems by the application of normal mode analysis.

In the Fryba book detailed solution of the problem of a constant force moving along infinite beam over an elastic foundation including its all possible speed and values of viscous damping is presented. The theory given in different papers has found widest field of application in calculating vibration level in large-span railway and highway bridges. Bridges of simply supported in most cases have a mass larger than the mass of vehicle and very low first natural frequencies. Because of this, if vibration of vehicle on own spring neglected, the effect of moving vehicle may approximately replaced by effects of moving forces.

A harmonic force moving along a simply supported beam was first solved by Timoshenko[6] and Inglis had worked out in detail [7].In the case of massless beam subjected to moving load, it is necessary to take the force effects of moving mass and d'alemberts principle of its inertia effects as well. This problem was originally solved and formulated by Wills [8].

The double Laplace transform has been used by Hamada [9] to find a solution for a beam with damping under the action of a moving mass less load. Hamada investigated the response problem of a simply supported and damped Euler–Bernoulli uniform beam of finite length traversed by a constant force moving at a constant speed by applying the double Laplace transformation with respect to time and the length coordinate along the beam. He obtained in closed form a particular solution for the dynamic deflection of the beam taken. The solution obtained is useful for computing beam stresses and also given that the forced vibration part can be derived in a double power series and that the coefficients of the series at the point of application of the moving force can be obtained by use of Bernoulli polynomials. As a numerical example simple approximate formulae obtained from the series are used to calculate the forced vibration parts of the deflection and the beam stresses at the mid-span of the beam when a moving load is exactly at the mid-point of the beam and errors are calculated.

The dynamic responses of a beam acted upon by moving loads or moving masses have been studied extensively in connection with the design of railway tracks and bridges and machining processes by Lee[10].The equation of motion in matrix form has been formulated for the dynamic response of a beam acted upon by a moving mass by using the Lagrangian approach and the assumed mode method and found that separation of the mass from the beam may occur for a relatively slow speed and small mass when the beam is clamped at both ends.

Michaltsos, Sophianopoulos, and Kounadis [11] studied the linear dynamic response of a simply supported uniform beam under a moving load of constant magnitude and velocity by including the effect of its mass. Using a series solution for the dynamic deflection in terms of normal modes the individual and coupling effect of the mass of the moving load of its velocity and of other parameters is completely verified and A variety of numerical results are given to draw important conclusions for structural design purpose.

Dynamic problem of a simply supported beam subjected to a constant force moving at a constant speed is analyzed by Olsson [12]. Analytical and finite element solutions to this fundamental moving load problem is shown and the results given by the author and other investigators are intended to give a basic understanding of the moving load problem and Some computational algorithms discussed.

Foda and Abduljabbar[13] designed a dynamic Green function approach that is used to determine the response of a simply supported Bernoulli-Euler beam of finite length subject to a moving mass traversing its span.

The proposed method produces a simple matrix expression for the deflection of the beam. The efficiency and simplicity of the method is illustrated by several numerical examples and the effect of various parameters on the dynamic response is investigated.

Michaltsos et al.[14] studied the linear dynamic response of a simply supported light bridge under a moving load or mass of constant magnitude and velocity including the effect of the centripetal and Carioles forces, which are always neglected. The individual and coupling effect of these forces in connection with the magnitude of the velocity of the moving load are fully discussed using a solution method based on an author’s older publication.

Mehri et al. [15] presented the linear dynamic response of uniform beams with different boundary conditions under the moving load based on the Euler-Bernoulli beam theory. Using a dynamic green function, effects of different boundary conditions, velocity of mass and some other parameters are studied and some of the numerical results are compared with those given in the literatures. An exact and direct modeling technique is formulated in this paper for forming beam structures with various boundary conditions, subjected to a mass moving at a constant speed. In order to validate the efficiency of the method, examples are given and results are compared with those available in the references. In addition to this the effect of a variation in the speed parameters of the system on the dynamic response of the beam was analyzed and the results were given in graphical and tabular form.

Ting et al. [16] studied and solved the problem using influence coefficient (static green function) and considering the distributed inertia effects of beam as external forces. At each place of load, numerical integration gas been done over whole length of beam. But using dynamic green function in the present study makes the deflection equation simple. Influence coefficients are the static deflection due to unit load which is used by Ting et al. in their investigation and given simple algorithms for moving mass problem.

Rieker and Trethewey[17] investigates the finite element analysis of an elastic beam structure under to a moving distributed load. The equations of motion are solved for a moving distributed mass train (mass per unit length) which is supposed to always stay in contact with the beam support structure. The finite element process is confirmed for some classes of moving mass problems (point force, a point mass, and a continuous load).

CHAPTER 3

DYNAMIC RESPONSE OF STRUCTURE

3. Dynamic response of structure.

The purpose of dynamic analysis is to know the structural behavior under the influence of various loads and to get the necessary information for design such as deformation, moments and dynamic forces etc. Structural analysis is classified in to static and dynamic. Static analysis deals with load which is time independent. But in dynamic analysis magnitude, direction and position of mass change with respect to time. Important dynamic loads for vibration analysis of bridge structure are vehicle motion and wave actions i.e. earthquake, stream flow and winds.

3.1 Simply supported beam structures

A system is assumed to be a Simply Supported beam system if it has a hinged connection at one end and roller connection in other end. Calculating the natural frequency of any system necessary to find how the system will act when just disturbed and left and to find out different form of excitation frequency to be avoided in the system. Vibration analysis of a Simply Supported beam system is essential as it can help us examine a number of real life systems. The following few examples are taken for a simply supported beam that helps for design changes accordingly for the most efficient systems. Now in the building of sky scrapers, transmission towers etc. the truss elements could be simply supported depending on their construction. Also in the design process of bridges simply supported beam analysis plays an significant role.

Fig1.Simply supported bridges and high way road.

In the automotive industry, the leaf spring suspension system study can be supposed to a simply supported beam analysis. In machines and machine tools, there are a number of systems which can be studied taking the supports to be simply supported.

The numerical theory on simply supported beam has many applications for the calculations related to long span railway and high way bridges. In most cases, the mass of the simply supported beam is higher than the mass of the vehicles. So that, if vibration of moving body on own springs is neglected, the effects of moving mass may more or less be replaced by the effects of moving forces.

In simply supported beam bridges, moments are not transfer through the support so their structural type is known as simply supported.

3.2 Cantilever beam structures

A beam with a laterally and rotationally fixed support at one end with no support (free) at the other end is called a cantilever beam. Cantilever construction allows for overhanging structures without external bracing. Cantilevers can also be constructed with trusses or slabs. Cantilever bridges are built with cantilevers, structures that project horizontally into the space and supported on only one end. For small bridges the cantilevers may be simple beams, though large cantilever bridges designed to handle the road or rail traffic use trusses.

The steel truss cantilever bridge are valuable as it can span distances of over 1,500 feet(460 m) and can be more simply constructed at difficult crossings by virtue of using little or no false work.

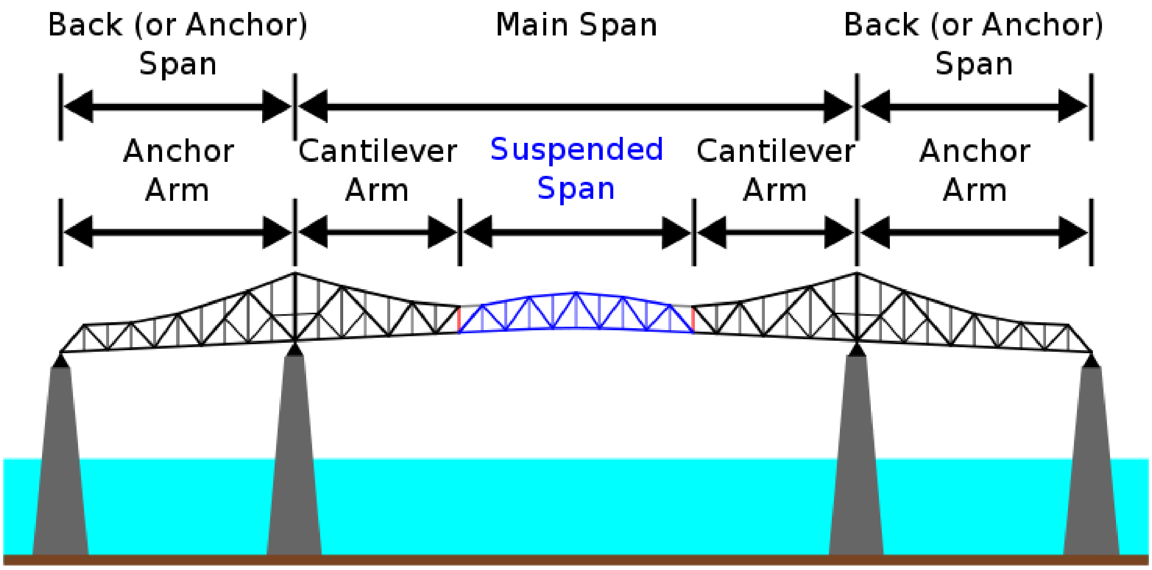

Fig2.cantilever beam structure

A simple cantilever span can be designed by two cantilever arms projecting from opposite sides of the obstacle to cross and meet at the center. In a common variant, the suspended span and the cantilever arms do not meet in the center instead of that they support a central truss bridge which rests on the ends of the cantilever arms. The suspended span can be built off-site and lifted into place or constructed in place using special traveling supports.

A common way to construct steel truss and pre-stressed concrete cantilever beam is to counterbalance each cantilever arm with another cantilever arm projecting the opposite direction which making a balanced cantilever.

In the recent years, tests are conducted on different types of beam bridges and the result obtained with more correctness with large displacements in the case of cantilever bridges than other types. For the behavior in dynamic condition of the cantilever beam, the analysis on frequency has to be made. Here the beam is assumed as a finite system with the entire mass of the beam mainly concentrates at 20 stations.

3.3 Analytical solution of the dynamic beam equation

3.3. a. Problem definition:

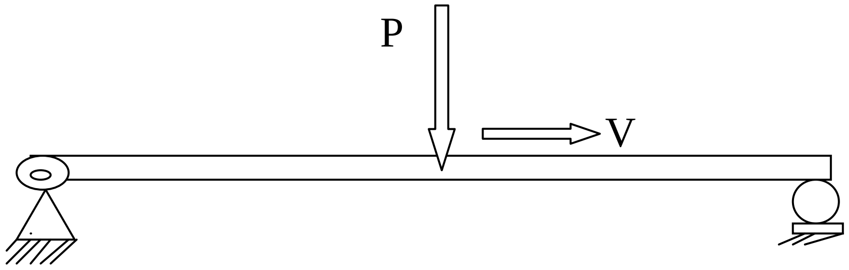

Fig.3 simply support beam subjected to moving mass.

The equation of motion of Euler-Bernoulli beam displacement can be formulated as [12]:

$$\rho A \frac{\partial^2 u_z}{\partial t^2} + EI_Y \frac{\partial^4 u_z}{\partial x^4} = \delta(x - vt)P \qquad (1)$$

Where, $u_z = u_z(x,t)$ is the beam deflection for $0 \leq x \leq L$ and $0 \leq t \leq {}^{L}/_{v}$

δ is the Dirac delta function.t is taken zero when moving force enters the beam.

Boundary condition and initial condition for simply supported condition are:

$$u_z(x,t) = \frac{\partial^2 u_z}{\partial x^2} = 0 \qquad at\ x = 0\ and\ L \qquad (2a)$$

$$u_z(x,0) = \frac{\partial u_z(x,0)}{\partial t} = 0 \qquad (2b)$$

Assumptions taken to solve the above equation are:

i) Structural assumptions are: a) initially straight beam

b) Linear elastic material

c) Small structural deformation

d) Damping effects is negligible

ii) Shear deformation and rotary inertia effects is neglected (Euler- Bernoulli beam),

Hence the height: length ratio of beam is small.

iii) Mass of the moving load is small compared to mass of beam, so we will consider

Only gravitational effects of load.

iv) Load moves at constant speed from left to right.

3.3. b Solving dynamic equation by separation of variable method[12]

$$u_z(x,t) = \sum_{n=1}^{\infty} y_n(t)\sin(n\pi x/L) \quad (3)$$

Where, $y_n(t)$ is modal displacement and $\sin n\pi x/L$ is modal function or Eigen functions.

Substituting equation (3) in equation (1), we get

$$\ddot{y}_n(t) + \omega_n^2 y_n(t) = (2P/\rho AL)\sin\overline{\omega}_n t \quad (4)$$

For n=1, 2, 3, 4 ….∞ and $0 \le t \le L/v$

$$\omega_n^2 = \frac{n^4\pi^4 EI}{\rho AL^4} \quad , \qquad \overline{\omega}_n = \frac{n\pi v}{L} \quad (5\ )$$

For n^{th}mode of vibration ω_n^2 *and* $\overline{\omega}_n$ *are* squared (circular) Eigen frequency and loading frequency respectively.

By using initial conditions,$y_n(0) = \dot{y}_n(0) = 0$ we got the solution of equation (4) as follows:

$$y_n(t) = (2P/\rho AL\omega_n^2)\left\{\frac{1}{(1-\beta_n^2)}\right\}(\sin\overline{\omega}_n t - \beta_n \sin\omega_n t), \qquad for\ \beta_n \ne 1 \qquad (6a)$$

Or

$$y_n(t) = (2P/\rho AL\omega_n^2) \times \tfrac{1}{2}(\sin\omega_n t - \omega_n t\ \cos\omega_n t), \qquad for\ \beta_n = 1 \qquad (6b)$$

Where β_n is the ratio of frequency

$$\beta_n = \frac{\overline{\omega}_n}{\omega_n} \quad (7)$$

Substituting equation (6) in equation (3) we obtained the series solution as

$$u_z(x,t) = u_{zs}(L/2)\frac{96}{\pi^4}\sum_{n=1}^{\infty}\left[\frac{1}{n^2(n^2-\alpha^2)}\left(\sin(n\pi t/\tau) - \frac{\alpha}{n}\sin\left(\frac{n^2\pi t}{\alpha}/\tau\right)\right)\sin(n\pi x/L)\right],$$

$$for\ \alpha \neq n \tag{8}$$

$$u_z(x,t) = u_{zs}(L/2)\frac{96}{\pi^4}\sum_{\substack{n=1\\ n\neq\alpha}}^{\infty}\left[\frac{1}{n^2(n^2-\alpha^2)}\left(\sin\left({}^{n\pi t}/_{\tau}\right) - \frac{\alpha}{n}\sin\left(\frac{n^2\pi t}{\alpha}/\tau\right)\right)\sin\left({}^{n\pi x}/_{L}\right)\right]$$
$$+ u_{zs}(L/2)\frac{96}{\pi^4}\left[\frac{1}{2\alpha^4}\left(\sin(\alpha\pi t/\tau) - {}^{\alpha\pi t}/_{\tau}\cos\left({}^{\alpha\pi t}/_{\tau}\right)\right)\sin\left({}^{\alpha\pi x}/_{L}\right)\right], \qquad \alpha = n$$

Here, $u_{zs}(L/2)$ is the static mid span displacement for the force P ,at mid-point

$$u_{zs}(L/2) = \frac{PL^3}{48EI}$$

$t = {}^{L}/_{v}$ (Traversing time for moving load) and α is a non-dimensional parameter and varies from 0 to 1.

3.3. c Dynamic equation by green function approach:

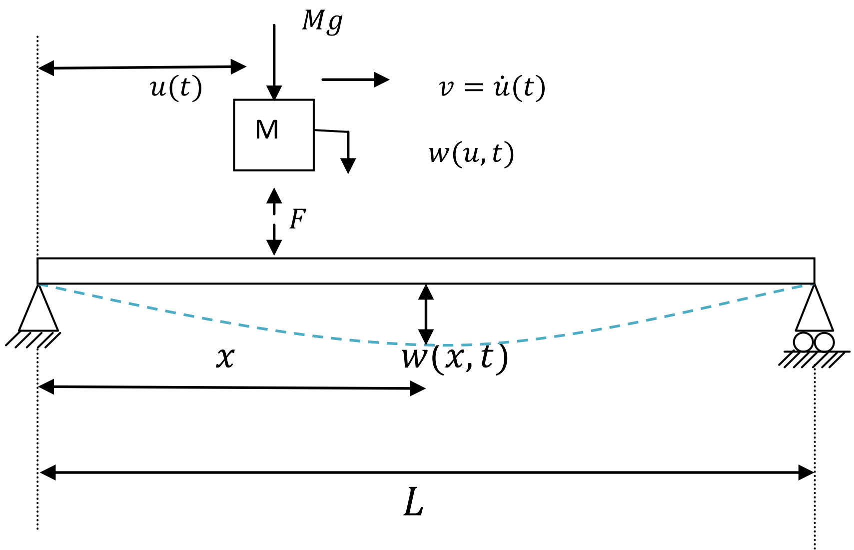

Fig.4 A mass traversing a beam with constant velocity

Fourth order differential equation of Euler-Bernoulli beam of finite length under a concentrated force [13].

$$EI\frac{\partial^4 w(x,t)}{\partial x^4} + m\frac{\partial^2 w(x,t)}{\partial t^2} = F\delta(x-u) \quad (9)$$

If we consider the inertia effect of mass, then the reaction force (F) exerted by mass M on the beam is given by Newton's second law which is:

$$F = M\left(g - \frac{d^2\beta}{dt^2}\right) \quad (10)$$

Here,β is the transverse effect of mass and g is acceleration due to gravity.

Hence, the particular solution of equation (9) for beam deflection at position x by the load at the position u is given by [13]:

$$w(x,u) = G(x,u)M\left[g - v^2\,\frac{d^2\beta(u)}{du^2}\right] \quad (11)$$

CHAPTER 4

EXPERIMENTAL WORK

4.1 Experimental Setup and Procedure:

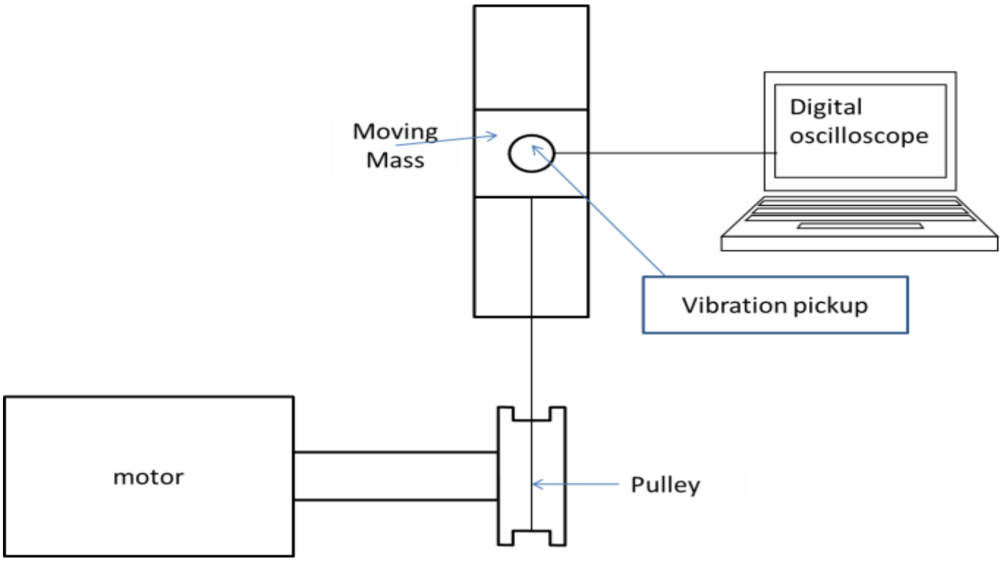

FIG 5. Experimental setup showing different equipment’s.

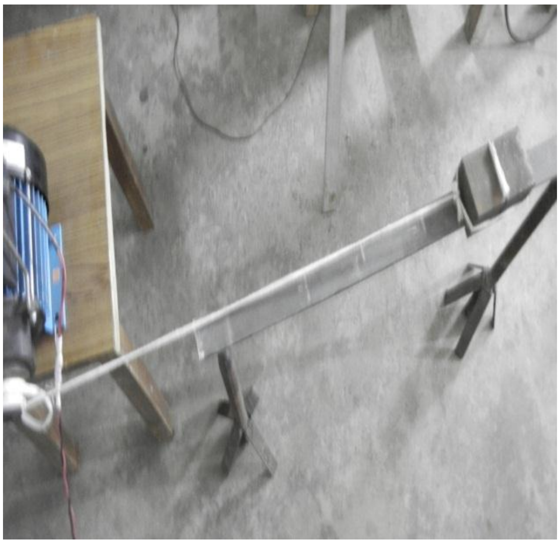

FIG.5(a, b) Experimental setup in dynamics lab at NIT ROURKELA For Simply support and Cantilever beam.

Experimental setup for a simply supported and cantilever beam is made which are shown in above figure. The beam of 1m is divided in to 20 stations and a vibration pickup is attached at lower the side of beam (attached at mid-span in case of simply supported and at the end of beam in cantilever case).vibration pickup is connected to the digital oscilloscope which shows the wave pattern generated on the screen. Amplitude of vibration or deflection of beam can be recorded from the oscilloscope. In simply supported beam amplitude of deflection at mid span is recorded while mass moving through different stations. But in case of cantilever beam deflection at end point of beam is recorded while mass traversing through different stations. Dynamic response of beam is studied for different speed and mass.

4.2 Equipment's Used

- vibration pick up or transducer
- digital oscilloscope(Tektronix 4000 series)
- U-shape iron block(used as moving mass)
- Aluminum and structural steel beams
- 0.5 HP motor with a pulley attached on shaft.

4.3 Equipment's Description

❖ Vibration pick up:

It is an electro-mechanical transducer which can able to convert the mechanical vibration generated into electrical signals which can be displayed on the screen of digital oscilloscope. Depending upon the nature of work vibration pickups are of different types i.e. accelerometer, displacement pick up, velocity picks up.

Fig6. vibration pick up used during experiment.

❖ Digital phosphor oscilloscope:

It’s an electronic device used to display the shape of electrical signals in the form of a graph in terms of voltage-time co-ordinate from which the amplitude of deflection of beam can be recorded. Some key features of the Tektronix 4000 series digital oscilloscope are:

- 1GHz,500MHz,and 350MHz bandwidths
- 2 and 4 channel models.
- 35000 waveforms/second display rate.
- USB and CompactFlash available for quick and easy storage.

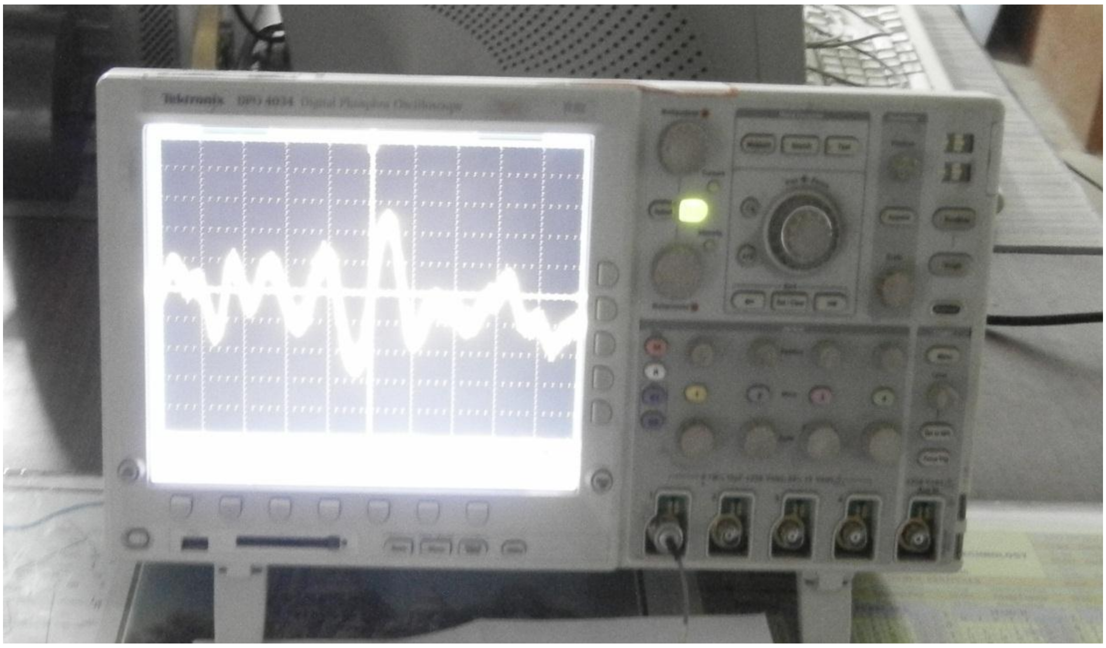

Fig.7 Tektronix 4000 series digital oscilloscope.

❖ U-shape iron block:

Two u-shapes iron block of 1.8kg and 0.9kg are made with the help of CNC machine in workshop of NIT Rourkela.

Fig.8 masses of 0.9kg and 1.8 kg used as moving masses.

Part program used is given below:

```
G00G53G90G40G80G15G69G94;
M03S2000;
G00G54X0Y0;
G00G43H1Z20;
G01Z0F500M08;
M98P2004L15;
G00G90Z50M09;
G00G53Y0;
M30;
G01G91Z-0.5F50;
G01G90X90X0;
G01X0Y0;
M99;
```

4.4. EXPERIMENTAL RESULTS

4.4. a. FOR SIMPLY SUPPORTED BEAM

Beam of 1m is divided in to 21 stations with each interval of 0.05m.

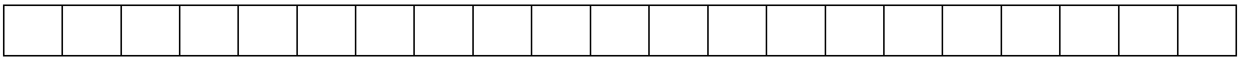

Fig. beam with 21 stations.

Case I: structural steel beam

1. Beam dimension and specification:

   Length (L) =1m, breadth (b) =5cm, width (d) =0.5cm

   E=200GPa

   $$I = \frac{bd^3}{12} = 5.2 \times 10^{-10} m^4$$

   Mass per unit length (m) =3kg/m

2. Moving mass(M): 0.9kg,1.8kg
3. Velocity of mass(v): 1, 2.5, 5,and 7 m/s

   Or corresponding velocity ratio (α) =0, 0.05, 0.125, 0.25, 0.375

   Here, $\alpha = {}^{v}/_{v_{cr}} \quad and \quad v_{cr} = \frac{\pi}{L}\sqrt{\frac{EI}{m}}$

Table for deflection of beam at mid span for moving mass traversing through different station and corresponding graphs are given below for different mass and speed.

x	y(α=0)	y(α=0.05)	y(α=0.125)	y(α=0.25)	y(α=0.375)
0	0	0	0	0	0
0.05	-0.1095	-0.1592	-0.1278	-0.1431	-0.1367
0.1	-0.296	-0.1953	-0.2919	-0.2809	-0.266
0.15	-0.3155	-0.4371	-0.439	-0.4355	-0.4198
0.2	-0.568	-0.5694	-0.4771	-0.5923	-0.588
0.25	-0.5875	-0.6898	-0.7021	-0.7413	-0.7625
0.3	-0.792	-0.5951	-0.9117	-0.8752	-0.9355
0.35	-0.8785	-0.8824	-0.8032	-0.9887	-1.0996
0.4	-0.944	-0.9486	-0.9732	-1.0767	-1.2464
0.45	-0.9855	-0.9905	-1.0178	-1.1344	-1.3661
0.5	-1.04	-1.0051	-1.0335	-1.117	-1.4473
0.55	-0.9855	-0.9905	-1.0179	-1.1407	-1.4773
0.6	-0.844	-0.9486	-0.9735	-1.0864	-1.447
0.65	-0.8785	-0.8824	-0.9036	-0.8974	-1.3482
0.7	-0.792	-0.7951	-0.8118	-0.8791	-1.1739
0.75	-0.6875	-0.5898	-0.7016	-0.7394	-0.7229
0.8	-0.568	-0.5694	-0.4666	-0.5892	-0.6108
0.85	-0.4365	-0.4372	-0.4403	-0.4397	-0.2927
0.9	-0.296	-0.2761	-0.2966	-0.2952	-0.0929
0.95	-0.1495	-0.1494	-0.1086	-0.1479	-0.1129
1	0	0	0	0	0

Table 1.deflection of structural steel beam at mid span for mass 0.9kg at different values of α.

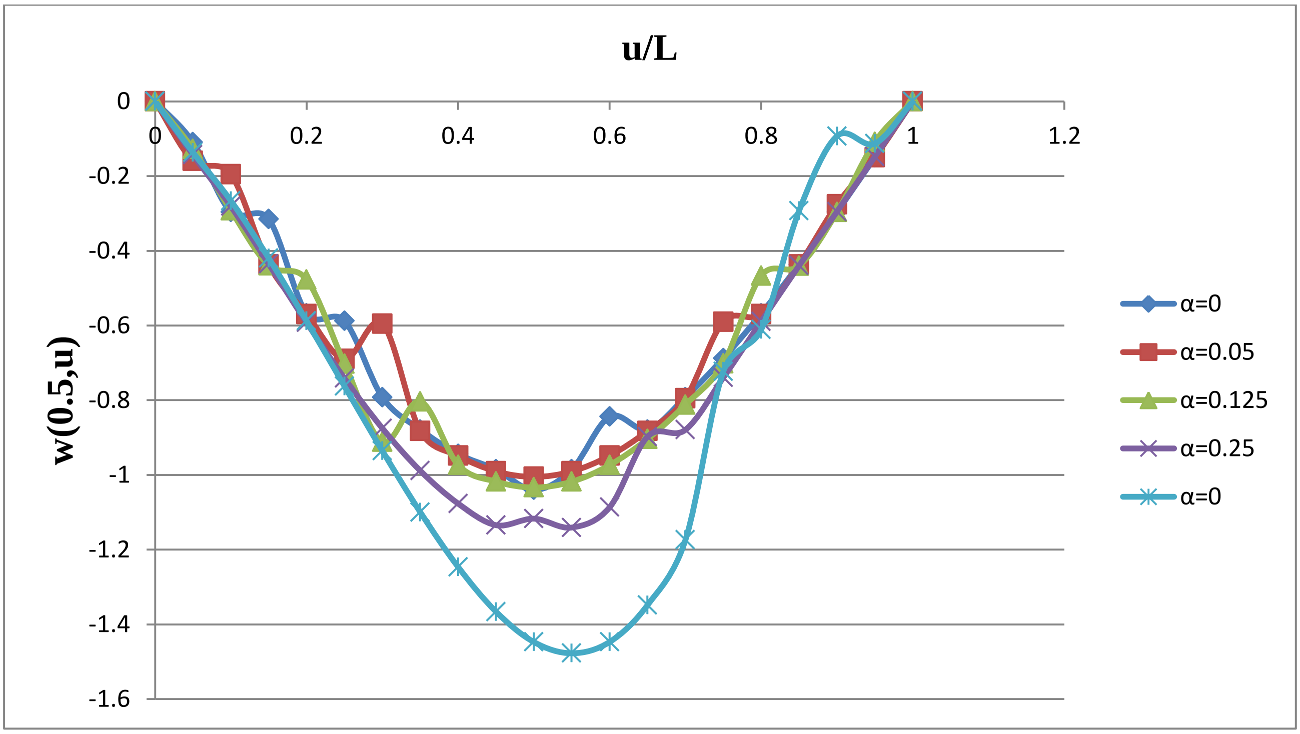

Fig.9 mid-span deflection of beam traversed by a moving mass when ${}^{M}/_{mL} = 0.3$

x	y(α=0)	y(α=0.25)	y(α=0.375)	y(α=0.05)	y(α=0.125)
0	0	0	0	0	0
0.05	-0.1295	-0.1196	-0.1122	-0.1486	-0.1439
0.1	-0.296	-0.2532	-0.2172	-0.2939	-0.2836
0.15	-0.2365	-0.399	-0.3463	-0.4365	-0.4322
0.2	-0.568	-0.5564	-0.4947	-0.5695	-0.5753
0.25	-0.7875	-0.7166	-0.6582	-0.6903	-0.6057
0.3	-0.792	-0.6718	-0.6329	-0.7962	-0.8197
0.35	-0.6785	-1.0142	-1.0145	-0.8841	-0.3147
0.4	-0.944	-1.1361	-1.1977	-0.9508	-0.99
0.45	-0.9855	-1.3286	-1.3753	-0.993	-1.035
0.5	-1	-1.2822	-1.5371	-1.0078	-1.0525
0.55	-0.9855	-1.2874	-1.67	-0.993	-1.0372
0.6	-0.944	-1.2396	-1.6614	-0.9508	-0.9905
0.65	-0.8785	-1.1364	-1.7951	-0.8841	-0.9161
0.7	-0.892	-0.9786	-1.747	-0.7963	-0.8186
0.75	-0.6875	-0.7734	-1.5815	-0.6903	-0.7032
0.8	-0.568	-0.5416	-1.0482	-0.5693	-0.7745
0.85	-0.4365	-0.3271	-0.7878	-0.4367	-0.4365
0.9	-0.296	-0.1951	0.0901	-0.2955	-0.2922
0.95	-0.2195	-0.1444	0.5866	-0.1489	-0.1453
1	0	0	0	0	0

TABLE.2. deflection of structural steel beam at mid span for mass 1.8kg at different values of α.

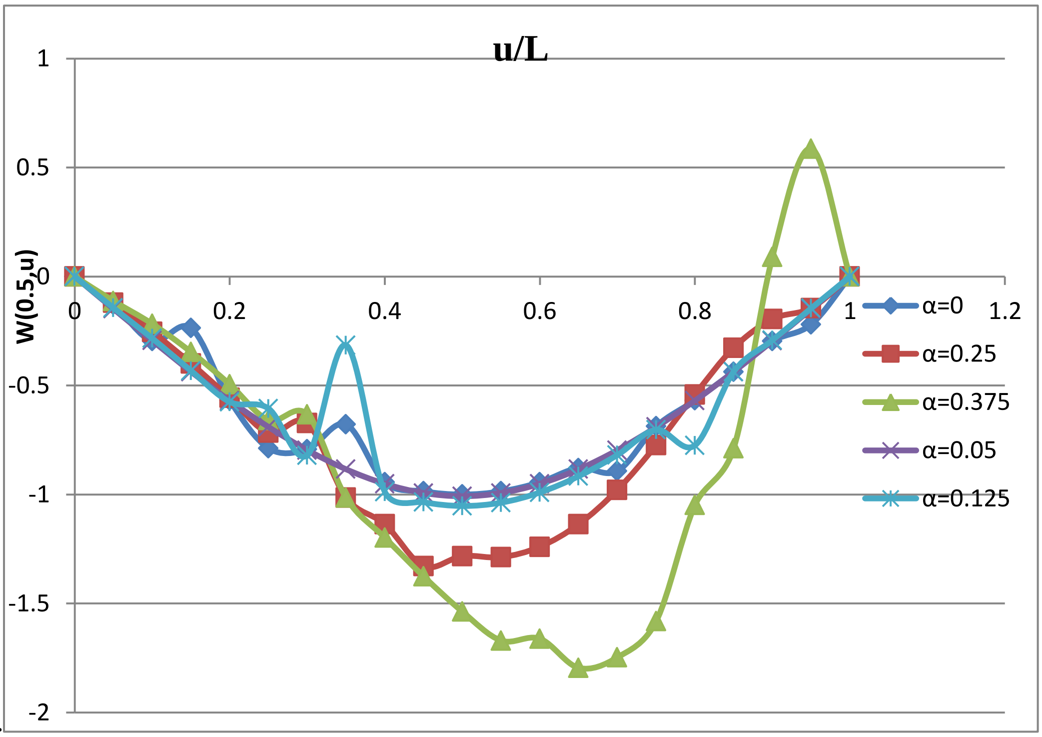

Fig.10 mid-span deflection of beam traversed by a moving mass when $^{M}/_{mL} = 0.6$

Case II: Aluminum beam

E(young's modulus)=69GPa

x	y(α=0)	y(α=0.05)	y(α=0.25)	y(α=0.125)	y(α=0.375)
0	0	0	0	0	0
0.05	-0.1495	-0.1092	-0.1431	-0.1478	-0.1167
0.1	-0.296	-0.2953	-0.2809	-0.2919	-0.266
0.15	-0.4365	-0.4371	-0.4355	-0.439	-0.4198
0.2	-0.568	-0.5694	-0.5923	-0.5771	-0.688
0.25	-0.5875	-0.6898	-0.7413	-0.7021	-0.7625
0.3	-0.792	-0.7951	-0.8752	-0.8117	-0.7355
0.35	-0.8785	-0.8824	-0.9887	-0.8032	-1.0996
0.4	-0.944	-0.9486	-1.1767	-0.9732	-1.2464
0.45	-0.8855	-0.9905	-1.1344	-1.0178	-1.1661
0.5	-1	-1.0051	-1.157	-1.0335	-1.4473
0.55	-0.9855	-0.9905	-1.1407	-1.0179	-1.4773
0.6	-0.944	-0.9486	-1.0864	-0.8735	-1.447
0.65	-0.8785	-0.8824	-0.9974	-0.9036	-1.3482
0.7	-0.792	-0.7951	-0.8791	-0.8118	-1.1739
0.75	-0.6875	-0.6898	-0.7394	-0.7016	-0.9229
0.8	-0.568	-0.5694	-0.5892	-0.4766	-0.5108
0.85	-0.4365	-0.4372	-0.4397	-0.4403	-0.2927
0.9	-0.296	-0.2961	-0.2952	-0.2966	-0.0929
0.95	-0.1495	-0.1494	-0.1479	-0.1486	-0.1129
1	0	0	0	0	0

TABLE.3. deflection of aluminum beam at mid span for mass 0.9kg for different values of α.

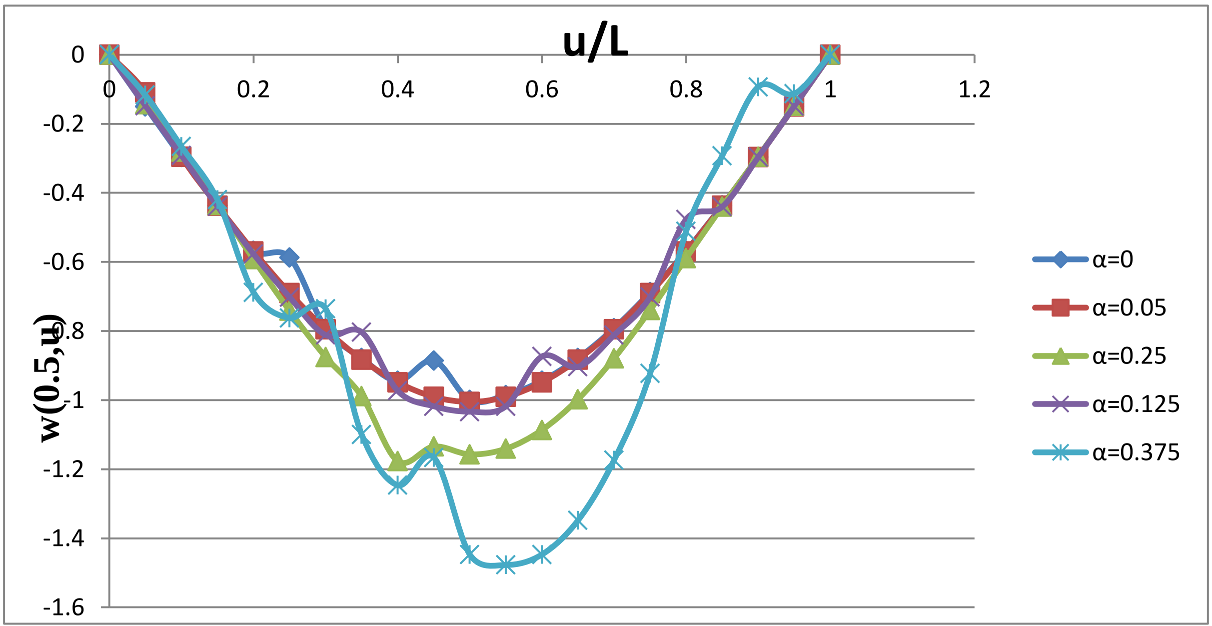

Fig.11 mid-span deflection of beam traversed by a moving mass when $M/_{mL} = 0.3$

x	y(α=0)	y(α=0.05)	y(α=0.25)	y(α=0.125)	y(α=0.375)
0	0	0	0	0	0
0.05	-0.1495	-0.1486	-0.1296	-0.1439	-0.1122
0.1	-0.296	-0.2939	-0.2532	-0.2836	-0.2172
0.15	-0.4365	-0.4365	-0.399	-0.4322	-0.5463
0.2	-0.568	-0.5695	-0.5564	-0.5753	-0.4947
0.25	-0.4875	-0.6903	-0.7166	-0.7057	-0.6582
0.3	-0.792	-0.7962	-0.6718	-0.8197	-0.8329
0.35	-0.8785	-0.8841	-1.0142	-0.9147	-1.0145
0.4	-0.944	-0.9508	-1.1361	-0.9876	-1.1977
0.45	-0.9855	-0.993	-1.2286	-1.035	-1.3753
0.5	-1	-1.1078	-1.2822	-1.0525	-1.5371
0.55	-0.9855	-0.993	-1.2874	-1.0372	-1.67
0.6	-0.944	-0.9508	-1.2396	-0.9905	-1.7614
0.65	-0.8785	-0.8841	-1.1364	-0.9161	-1.7951
0.7	-0.792	-0.7963	-0.9786	-0.8186	-1.647
0.75	-0.6875	-0.4903	-0.7734	-0.7032	-1.5815
0.8	-0.568	-0.5693	-0.5416	-0.5745	-1.0482
0.85	-0.4365	-0.4367	-0.3271	-0.2365	-0.4878
0.9	-0.296	-0.2955	-0.1951	-0.2922	0.0901
0.95	-0.1495	-0.1489	-0.1444	-0.1453	0.1866
1	0	0	0	0	0

TABLE.4. deflection of aluminum beam at mid span for mass 1.8kg.

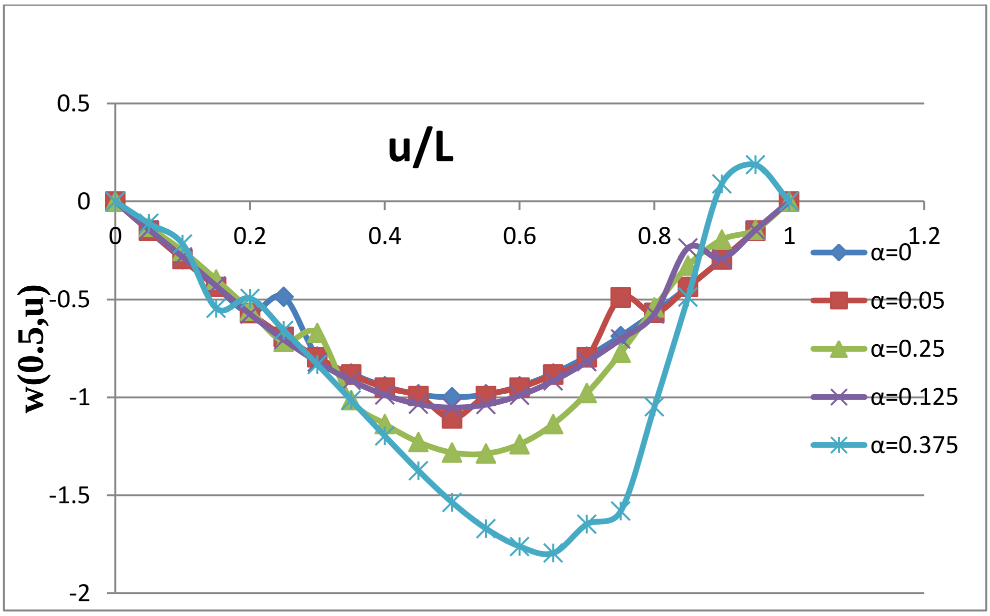

Fig12.Mid-span deflection of beam traversed by a moving mass when $M/_{mL} = 0.6$

4.4. b. For Cantilever Beam(structural steel)

Beam is divided in to five stations with each interval of 0.2m.

For structural steel and mass of 0.9 kg

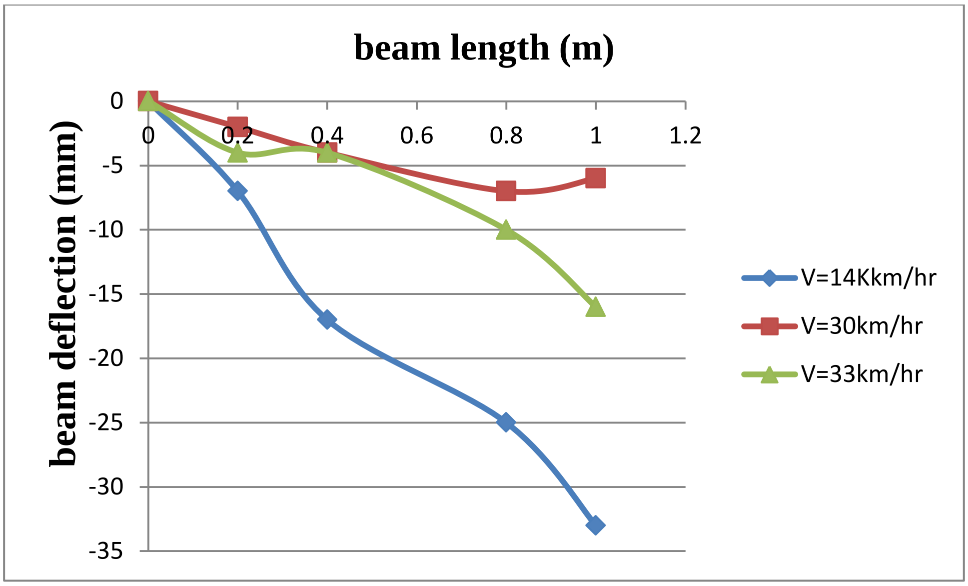

Fig.13 deflection of cantilever beam at end point for different velocities.

For the mass M=1.8kg

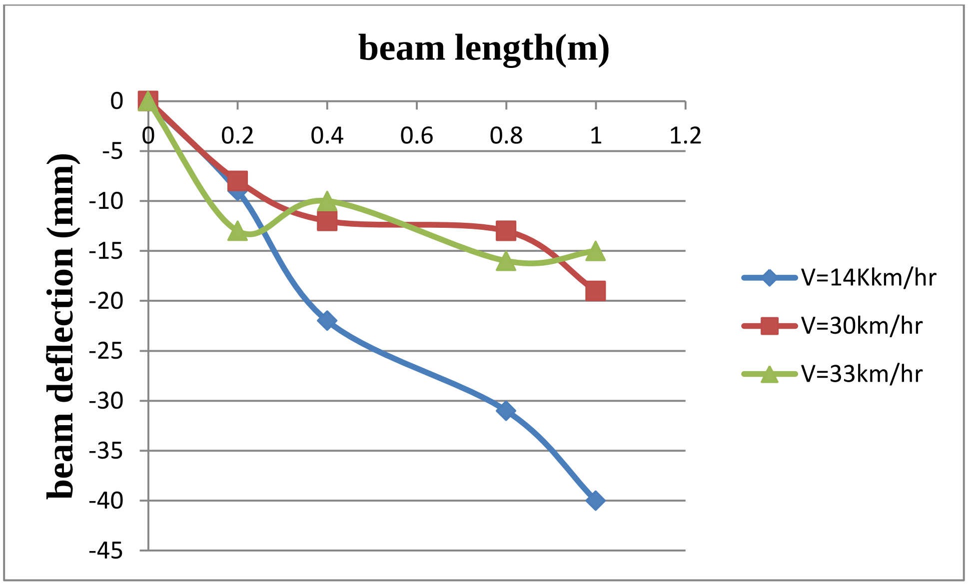

Fig.14 deflection of cantilever beam at end point for different velocities for mass M=1.8kg

DISCUSSION

An experimental work is done for dynamic response of both cantilever and simply supported beam subjected to moving mass. Experimental results have been presented for deflection of beam at a point with respect to position of mass on beam. The following inferences are given from the experimental analysis:

a) In simply supported case, the result shows that the maximum deflection of beam increasing with increase in velocity of mass and also position of maximum deflection deviates from mid-span of beam. Maximum deflection of beam increase with increase in weight of moving mass.
b) In cantilever beam the result shows that deflection of beam at free end decrease with increase in velocity of mass.This is due to the fact that, at higher velocity of mass, lower modes are not excited which always contribute to large deflection of beam. Vibration at lower mode gives larger amplitude as in comparison to higher mode resulting in the decrease in dynamic deflection. End point deflection increases with increase in mass of moving body because of increase in inertia of load.

CONCLUSION

Dynamic analysis of a simply support and cantilever beam subjected to moving mass at constant speed is investigated. The influence of variation of parameters i.e. velocity of mass and increase in weight of moving mass on dynamic response is studied. From the above results and discussion it can be concluded that the position of maximum deflection of beam occurs far from mid span. Maximum deflection of beam occurs close to end point of beam at very high velocity of mass. The dynamic response of beam is more influenced by change in speed of mass as compared to change in mass ratio of system. The effect of changing the material on dynamic behavior of beam for both cantilever and simply support is same.

REFERENCES

[1] R.Willis, ‘Report of the Commissioners Appointed to Inquire into the Application of Iron to Railway Structures’ London: Stationary office, 1949

[2] L. Fryba ‘Vibration of Solids and Structures under Moving Loads. Groningen’, Noordhoff International, 1972

[3] G. G. Stokes.’Discussions of a differential equation relating to the breaking of railway bridges’ Mathematical and Physical Papers 2, 1883, 178-220.

[4] Kolousek V.: Dynamics of Civil Engineering Structures. Part I — General Problems, second edition; Part II — Continuous Beams and Frame Systems, second edition; Part III — Selected Topics (in Czech.) SNTL, Prague 1967, 1956, 1961.

[5] Kolousek v., Civil engineering structures subjected to dynamic loads. SVTL Bratislava 1967. In Slovak.

[6] Timoshenko S.P.: On the forced vibration of bridges. Philosophy Magazine, ser. 6, 43 (1922),1018.

[7] Inglish C.E.:‘A Mathematical Treatise on vibration in railway bridges’ The university press, Cambridge, 1934.

[8] Willis R. et al.: Preliminary essay to the appendix B.: Experiments for determining the effects produced by causing weights to travel over bars with different velocities. In: Grey G. et al.: Report of the Commissioners Appointed to inquire into the application of iron to railway structures. W.Clowes and Sons, London 1849. Reprinted in: Barlow P.: Treatise on the strength of Timber, Cast Iron and Malleable Iron. London 1851.

[9] T.R.Hamada,’Dynamic analysis of a beam under a moving force: a double Laplace transforms solution’, Journal of Sound and Vibration 74, (1981), 221-233.

[10]H.P.Lee,’Dynamic response of a beam with a moving mass’,Journal of Sound and Vibration 191, (1996), 298-294.

[11] G.Michaltsos, D. Sopianopoulo and A. N. Kounadis’The effect of moving mass and other parameters on the dynamic response of a simply supported beam’, Journal of Sound and Vibration 191 (1996),357-362.

[12] M.Olsson,’On the fundamental moving mass problem’, Journal of Sound and Vibration 145 (1991), 299-307

[13] M. A Foda and Z. Abduljabbar,’A Dynamic Green Function Formulation for the Response of a Beam Structure to a Moving Mass’, Journal of sound and vibration,210(3),1998,295-306.

[14] G.T. Michaltsos and A.N. Kounadis,'The Effects of Centripetal and Coriolis Forces on the Dynamic Response of Light(steel)Bridges under Moving Loads', journal of vibration and control,7(3),2001,315-326.

[15] B. Mehri, A. Davar and O. Rahmani 'Dynamic Green Function Solution of Beams under a Moving Load with Different Boundary Conditions',Scientia iranica, 16(3,)2009,273-279.

[16] E. C.Ting, J. Genin and J. H. Ginsberg 'A general algorithm for moving mass problems',Journal of Sound and Vibration, 33, 1974, 49-58.

[17]J. R. Rieker and M. W. Trethewey,'Finite Element Analysis of an Elastic Beam Structure Subjected to a Moving Distributed Mass Train', Mechanical Systems and Signal Processing 13 (1999) 31-51.